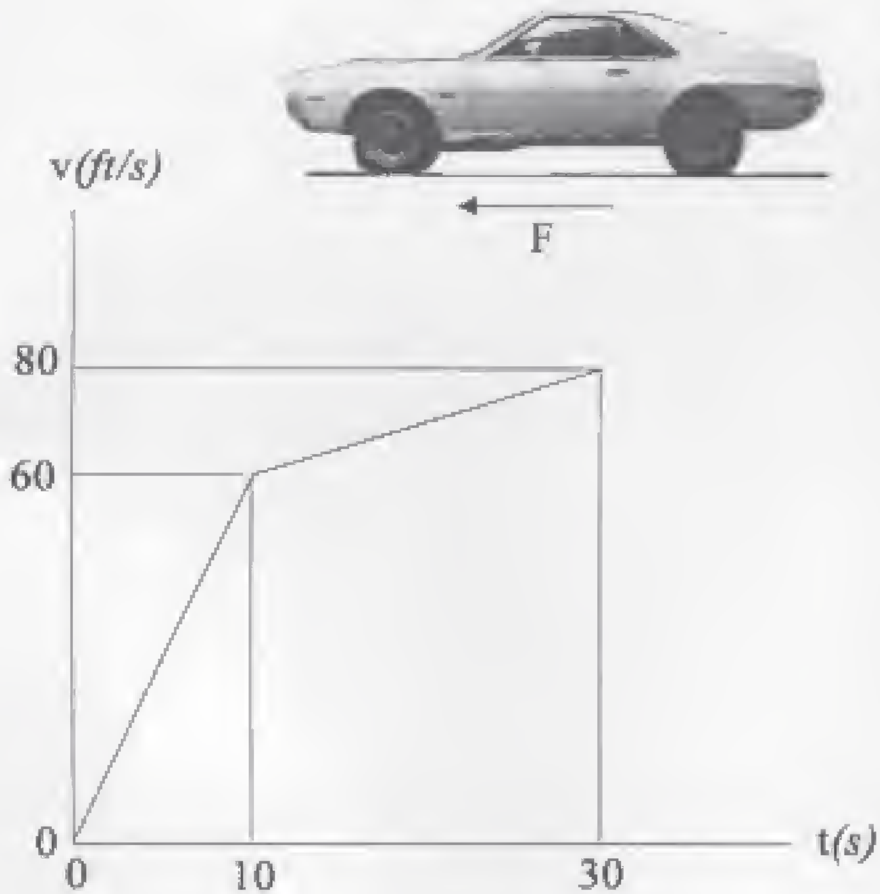
Question 1 *(20 points)*

The speed of the 3500-lb sports car is plotted over the 30-s time period shown below.

a) Plot the variation of the traction force **F** needed to cause the motion

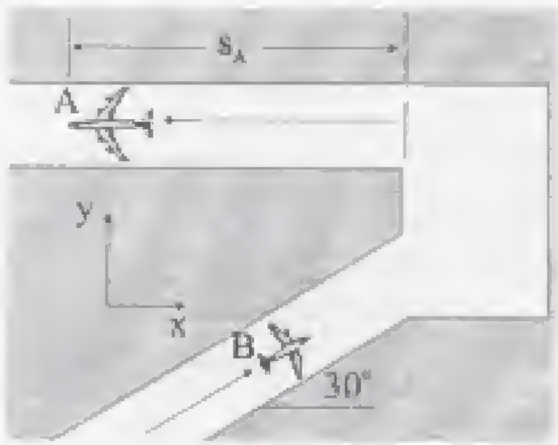
b) At the end of the 30-s time period, the brakes are applied and the car is stopped in a distance of 164 ft. If it is known that all four wheels contribute equally to the braking force, determine the braking force $\mathbf{F}_B$ at each wheel. Assume a constant deceleration and that the weight of the car is distributed evenly over all four tires.



Question 2 *(20 points)*

The 300–Mg research jet **A** has three engines, each of which produce an approximately constant thrust of 240 kN during the takeoff roll. A small commuter aircraft **B** taxis toward the end of the runway at a constant speed $v_B = 30$ km/h as shown below.

(a) Determine the velocity and acceleration, which the jet **A** appears to have relative to a pilot observer in the small aircraft **B** 10 seconds after **A** begins its takeoff roll *(expressed as a magnitude and direction)*. Determine also the minimum length s_A of the horizontal runway required if the takeoff speed of the jet A is 220 km/h. Neglect air and rolling resistance.



(b) The research jet **A** travels some distance after takeoff before firing a rocket shown below in a vertical plane. At the instant considered the rocket has a mass of 2000 kg and is propelled by a thrust force **T** of 32 kN. The rocket is also subjected to atmospheric resistance **R** of 9.6 kN. If the rocket has a velocity of 3 km/s and if the gravitational acceleration **g** is 6 m/s^2 at the altitude of the rocket, calculate the the radius of curvature ρ of its path for the position described and the time-rate-of-change of the velocity of the rocket.

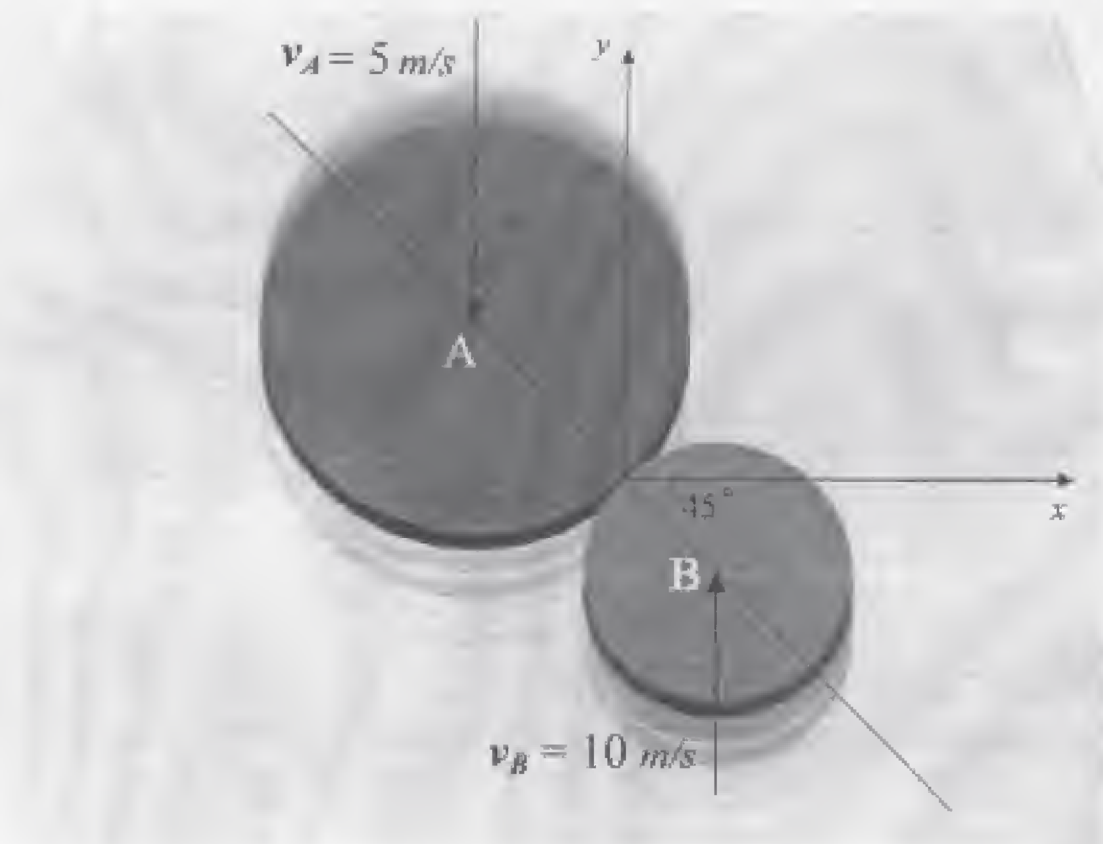
Please Change Exam Booklet now

Question 3 ***(20 points)***

Discs A and B travel on a smooth surface at a velocity of 5 m/s and 10 m/s, respectively. The mass of disc A is 20 kg while the mass of disc B is 4 kg. If they collide as shown find:

a) The speed of both discs after impact, assuming that the coefficient of restitution is 0.9.

b) Using information found in part (a) and given that the impact occurs in 0.005 seconds find the magnitude of the average impulsive force on disc A

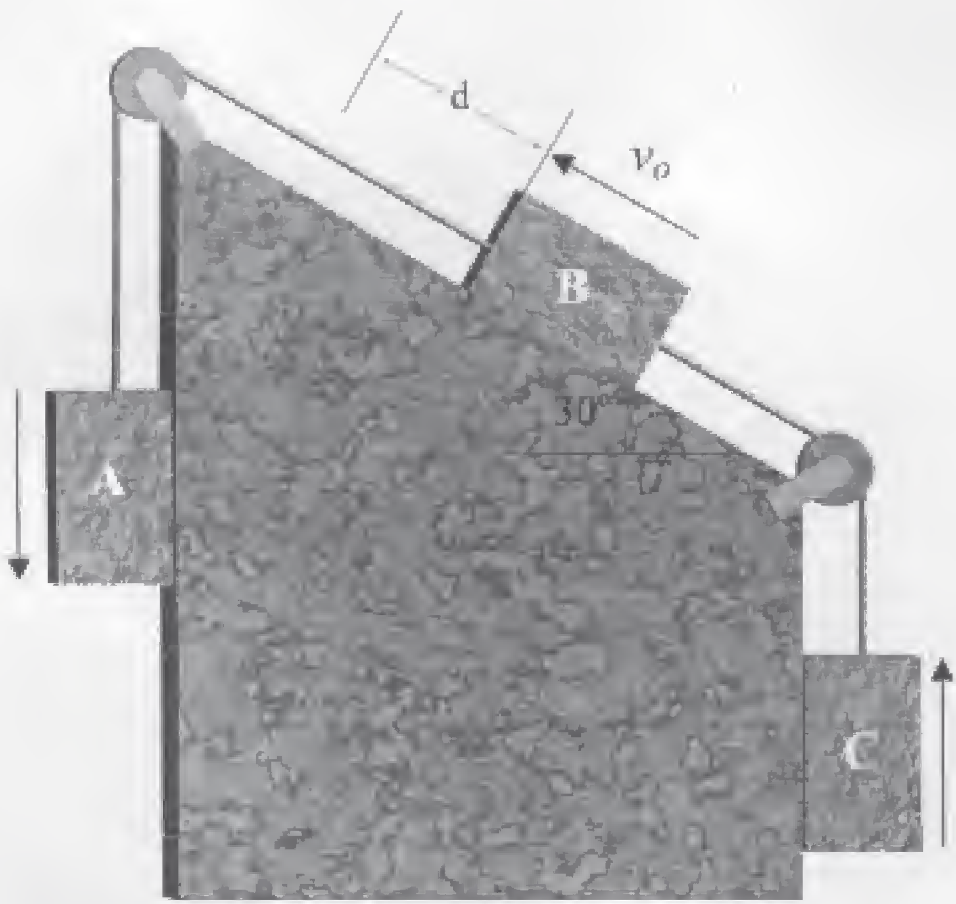


Question 4 *(20 points)*

Blocks A, B and C have an initial velocity of 2 m/s as shown in the diagram. If the mass of block A is 3 kg, the mass of blocks B and C is 2 kg each, and the coefficient of kinetic friction μ_k is 0.1, determine:

a) the distance block B travels before coming to rest
b) the minimum friction force required to ensure the blocks stay at rest
c) If the cables connecting blocks are cut, and assuming block B slides down the incline, find the power lost to friction of block B when it reaches the original starting position in (a).

Assume the vertical surfaces of the base are smooth



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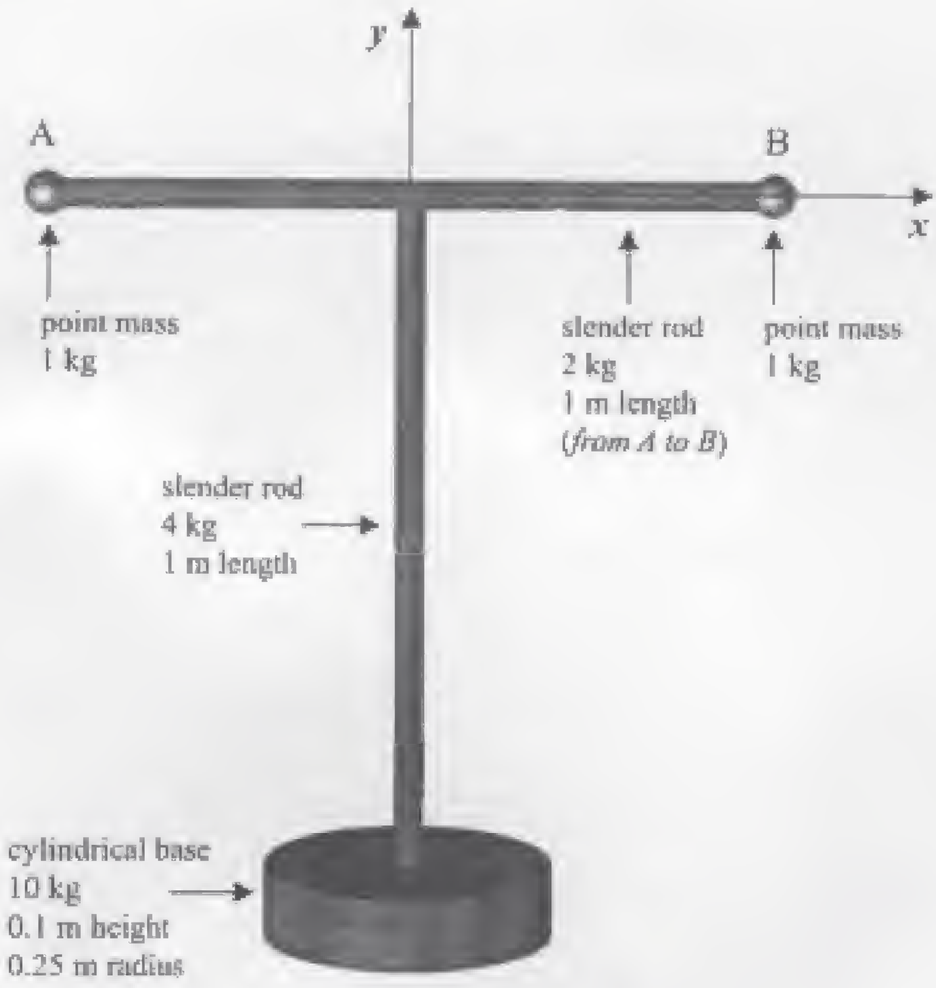
Question 5 *(20 points)*

For the following tamping tool, find:

a) The mass moment of inertia about the x-axis and the mass moment of inertia about the y-axis.

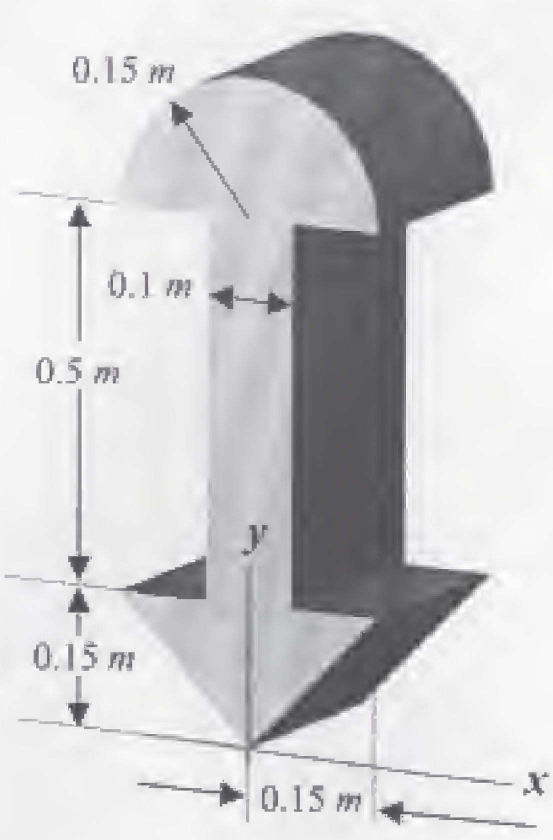
b) The location of the center of gravity of the tool. Give both the x and y coordinates.

c) The mass moment of inertia about an axis parallel to the x-axis and running through center of gravity.



Question 6 *(20 points)*
Given the following beam cross section, determine:

a) The y-coordinate of the centroid
b) The area moment of inertia about an axis parallel to the x-axis and running through the centroid



G E 125 – *Formula Sheet*

Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$ $\quad a_t = \dot{v} = v\frac{dv}{ds}$

$a_n = \frac{v^2}{\rho} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant $\alpha = \alpha_c$*
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $\quad I = \int r^2\,dm$

Parallel-Axis Theorem $\quad I = I_G + md^2$

Radius of Gyration $\quad k = \sqrt{\frac{I}{m}}$

Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \frac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \quad \varepsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $\quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

$$T_1 + V_1^g + V_1^e + \sum U_{1\to 2} = T_2 + V_2^g + V_2^e$$

$$\bar{x} = \frac{\sum \tilde{x}m}{\sum m} \qquad \bar{y} = \frac{\sum \tilde{y}m}{\sum m} \qquad \bar{z} = \frac{\sum \tilde{z}m}{\sum m}$$

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A} \qquad \bar{z} = \frac{\sum \tilde{z}A}{\sum A}$$

$$I_x = \bar{I}_{x'} + Ad_y^2 \qquad I_y = \bar{I}_{y'} + Ad_x^2 \qquad I = I_G + md^2$$

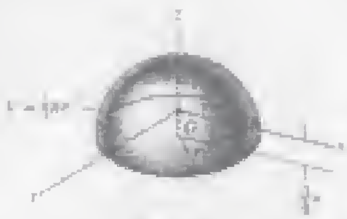
Center of Gravity and Mass Moment of Inertia of Homogeneous Solids

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$

Cylinder

$I_{xx} = I_{yy} = \frac{1}{12}m(3r^2 + h^2)$ $I_{zz} = \frac{1}{2}mr^2$

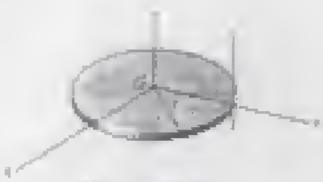


Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2$ $I_{zz} = \frac{2}{5}mr^2$

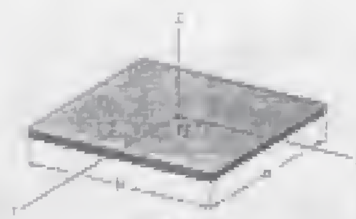
Cone

$I_{xx} = I_{yy} = \frac{3}{80}m(4r^2 + h^2)$ $I_{zz} = \frac{3}{10}mr^2$



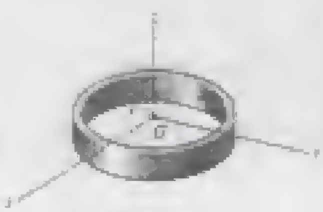
Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4}mr^2$ $I_{zz} = \frac{1}{2}mr^2$ $I_{z'z'} = \frac{3}{2}mr^2$



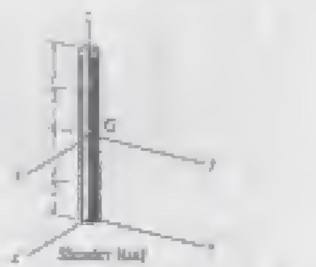
Thin plate

$I_{xx} = \frac{1}{12}mb^2$ $I_{yy} = \frac{1}{12}ma^2$ $I_{zz} = \frac{1}{12}m(a^2 + b^2)$



Thin ring

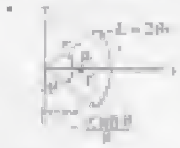
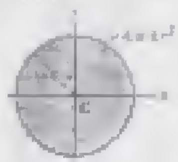
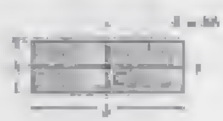
$I_{xx} = I_{yy} = \frac{1}{2}mr^2$ $I_{zz} = mr^2$



Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12}ml^2$ $I_{x'x'} = I_{y'y'} = \frac{1}{3}ml^2$ $I_{zz} = 0$

Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia
 Circular arc segment	 Circular sector area	
 Quarter and semicircle arcs	 Quarter circle area	
 Trapezoidal area	 $\frac{4r}{3\pi}$ Semicircular area	$= \frac{1}{8}\pi r^4$ $= \frac{1}{8}\pi r^4$
 Semiparabolic area	 Circular area	
 Exponential area	 Rectangular area	$= \frac{1}{12} bh^3$ $= \frac{1}{12} hb^3$
 Parabolic area	 $\frac{1}{3}h$ Triangular area	$= \frac{1}{36} bh^3$

Integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) + C$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ba}}\ln\left[\frac{a+x\sqrt{-ab}}{a-x\sqrt{-ab}}\right] + C, \quad ab<0$$

$$\int \frac{x\,dx}{a+bx^2} = \frac{1}{2b}\ln(bx^2+a) + C$$

$$\int \frac{x^2\,dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b\sqrt{ab}}\tan^{-1}\frac{x\sqrt{ab}}{a} + C, \quad ab>0$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left[\frac{a+x}{a-x}\right] + C, \quad a^2>x^2$$

$$\int \sqrt{a+bx}\,dx = \frac{2}{3b}\sqrt{(a+bx)^3} + C$$

$$\int x\sqrt{a+bx}\,dx = \frac{-2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$$

$$\int x^2\sqrt{a+bx}\,dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right] + C, \quad a>0$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C$$

$$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3}$$

$$+ \frac{a^2}{8}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right) + C, \quad a>0$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x+\sqrt{x^2 \pm a^2}\right)\right] + C$$

$$\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2 \pm a^2)^3} + C$$

$$\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2 \pm a^2}$$

$$- \frac{a^4}{8}\ln\left(x+\sqrt{x^2 \pm a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}}\ln\left[\sqrt{a+bx+cx^2}\right.$$

$$\left. + x\sqrt{c} + \frac{b}{2\sqrt{c}}\right] + C, \quad c>0$$

$$= \frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{-2cx-b}{\sqrt{b^2-4ac}}\right) + C, \quad c>0$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax) + C$$

$$\int x^2\cos(ax)\,dx = \frac{2x}{a^2}\cos(ax)$$

$$+ \frac{a^2x^2-2}{a^3}\sin(ax) + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

APPENDIX A

Mathematical Expressions

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$, $\quad \cosh x = \dfrac{e^x + e^{-x}}{2}$, $\quad \tanh x = \dfrac{\sinh x}{\cosh x}$

Trigonometric Identities

$\sin\theta = \dfrac{A}{C}$, $\quad \csc\theta = \dfrac{C}{A}$

$\cos\theta = \dfrac{B}{C}$, $\quad \sec\theta = \dfrac{C}{B}$

$\tan\theta = \dfrac{A}{B}$, $\quad \cot\theta = \dfrac{B}{A}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos\theta = \pm\sqrt{\dfrac{1 + \cos 2\theta}{2}}$, $\quad \sin\theta = \pm\sqrt{\dfrac{1 - \cos 2\theta}{2}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta = \sec^2\theta \qquad\qquad 1 + \cot^2\theta = \csc^2\theta$

Power-Series Expansions

$\sin x = x - \dfrac{x^3}{3!} + \cdots \qquad\qquad \sinh x = x + \dfrac{x^3}{3!} + \cdots$

$\cos x = 1 - \dfrac{x^2}{2!} + \cdots \qquad\qquad \cosh x = 1 + \dfrac{x^2}{2!} + \cdots$

Derivatives

$\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}(\cot u) = -\csc^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sec u) = \tan u \sec u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\csc u) = -\csc u \cot u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sin u) = \cos u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cos u) = -\sin u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\tan u) = \sec^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sinh u) = \cosh u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cosh u) = \sinh u\dfrac{du}{dx}$